

# Mathematica 11.3 Integration Test Results

Test results for the 50 problems in "Charlwood Problems.m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int -\text{ArcSin}[\sqrt{x} - \sqrt{1+x}] dx$$

Optimal (type 3, 69 leaves, ? steps):

$$\frac{(\sqrt{x} + 3\sqrt{1+x})\sqrt{-x+\sqrt{x}}\sqrt{1+x}}{4\sqrt{2}} - \left(\frac{3}{8} + x\right)\text{ArcSin}[\sqrt{x} - \sqrt{1+x}]$$

Result (type 3, 205 leaves):

$$-x\text{ArcSin}[\sqrt{x} - \sqrt{1+x}] - \left( (1+x) (1+2x-2\sqrt{x}\sqrt{1+x})^2 \right. \\ \left. \left( 2\sqrt{-x+\sqrt{x}}\sqrt{1+x} (-3-2x+2\sqrt{x}\sqrt{1+x}) + 3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}} \text{Log}[2\sqrt{-x+\sqrt{x}}\sqrt{1+x} + \sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}] \right) \right) / \\ \left( 8\sqrt{2} (-\sqrt{x} + \sqrt{1+x})^3 (1+x - \sqrt{x}\sqrt{1+x})^2 \right)$$

Problem 5: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x]^2}{\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}} dx$$

Optimal (type 3, 45 leaves, ? steps):

$$\frac{x}{3} + \frac{1}{3}\text{ArcTan}\left[\frac{\text{Cos}[x](1+\text{Cos}[x]^2)\text{Sin}[x]}{1+\text{Cos}[x]^2\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\right]$$

Result (type 4, 159 leaves):

$$\frac{2 i \cos [x]^2 \operatorname{EllipticPi}\left[\frac{3}{2} + \frac{i\sqrt{3}}{2}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{3}}} \tan [x]\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right] \sqrt{1-\frac{2 i \tan [x]^2}{-3 i+\sqrt{3}}} \sqrt{1+\frac{2 i \tan [x]^2}{3 i+\sqrt{3}}}}{\sqrt{-\frac{i}{-3 i+\sqrt{3}}} \sqrt{15+8 \cos [2 x]+\cos [4 x]}}$$

**Problem 6: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \tan [x] \sqrt{1+\tan [x]^4} dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$-\frac{1}{2} \operatorname{ArcSinh}[\tan [x]^2] - \frac{\operatorname{ArcTanh}\left[\frac{1-\tan [x]^2}{\sqrt{2} \sqrt{1+\tan [x]^4}}\right]}{\sqrt{2}} + \frac{1}{2} \sqrt{1+\tan [x]^4}$$

Result (type 4, 7083 leaves):

$$\frac{1}{2} \sqrt{1+\tan [x]^4} -$$

$$\left(4 \cos [x]^2 \left( (2+6 i) - \frac{8}{\sqrt{-1-i}} - 5 \sqrt{-1+i} + (2+4 i) \sqrt{2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(2 i+\sqrt{-1-i}+\sqrt{-1+i})\left((-1-2 i)+2 \sqrt{-1+i}+\tan \left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2 \sqrt{-1-i}+\tan \left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right], 4-2 \sqrt{2}\right] +$$

$$\left( (-4-4 i) - (3-5 i) \sqrt{-1-i} + (5-3 i) \sqrt{-1+i} + 4(-1-i)^{3/2} \sqrt{-1+i} \right)$$

$$\operatorname{EllipticPi}\left[\frac{2 \sqrt{-1+i}\left((-1+i)+\sqrt{-1-i}\right)}{\left((-1-i)+\sqrt{-1+i}\right)\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(2 i+\sqrt{-1-i}+\sqrt{-1+i})\left((-1-2 i)+2 \sqrt{-1+i}+\tan \left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2 \sqrt{-1-i}+\tan \left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right], 4-2 \sqrt{2}\right] +$$

$$\left( (-4-4 i) - (1-4 i) \sqrt{-1-i} + (4-i) \sqrt{-1+i} + 2(-1-i)^{3/2} \sqrt{-1+i} \right)$$

$$\operatorname{EllipticPi}\left[\frac{2 \sqrt{-1+i}\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(2 i+\sqrt{-1-i}+\sqrt{-1+i})\left((-1-2 i)+2 \sqrt{-1+i}+\tan \left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2 \sqrt{-1-i}+\tan \left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right], 4-2 \sqrt{2}\right]$$

$$\sqrt{\frac{(2i + \sqrt{-1-i} - \sqrt{-1+i}) \left( (1-2i) + 2\sqrt{-1-i} - \tan\left[\frac{x}{2}\right]^2 \right)}{(-2i + \sqrt{-1-i} + \sqrt{-1+i}) \left( (-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2 \right)}} \left( (-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2 \right)^2$$

$$\sqrt{-\frac{(1-i) \left( (-1+2i) + 2\sqrt{-1-i} \right) \left( 1 - (2+4i) \tan\left[\frac{x}{2}\right]^2 + \tan\left[\frac{x}{2}\right]^4 \right)}{\left( (-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2 \right)^2} \left( \frac{2 \sec[x] \sin[3x]}{\sqrt{3+\cos[4x]}} - \frac{2 \tan[x]}{\sqrt{3+\cos[4x]}} \right) \sqrt{1+\tan[x]^4}} \right)$$

$$\left( \sqrt{-1+i} \left( (-12+4i) + (7+8i) \sqrt{-1-i} \right) \left( (2+2i) - (2-i) \sqrt{-1+i} \right) \sqrt{3+\cos[4x]} \right)$$

$$\left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^2 \sqrt{\frac{1 - 4 \tan\left[\frac{x}{2}\right]^2 + 22 \tan\left[\frac{x}{2}\right]^4 - 4 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{\left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^4}}$$

$$\left( - \left( \left( (2+6i) - \frac{8}{\sqrt{-1-i}} - 5\sqrt{-1+i} + (2+4i)\sqrt{2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(2i+\sqrt{-1-i}+\sqrt{-1+i}) \left( (-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2 \right)}}{\sqrt{-1+i} \left( (-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2 \right)}}\right], 4-2\sqrt{2} \right] + \right.$$

$$\left. \left( (-4-4i) - (3-5i)\sqrt{-1-i} + (5-3i)\sqrt{-1+i} + 4(-1-i)^{3/2}\sqrt{-1+i} \right) \right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i} \left( (-1+i) + \sqrt{-1-i} \right)}{\left( (-1-i) + \sqrt{-1+i} \right) \left( 2i + \sqrt{-1-i} + \sqrt{-1+i} \right)}, \text{ArcSin}\left[\frac{\sqrt{(2i+\sqrt{-1-i}+\sqrt{-1+i}) \left( (-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2 \right)}}{\sqrt{-1+i} \left( (-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2 \right)}}\right], 4-2\sqrt{2} \right] +$$

$$\left( (-4-4i) - (1-4i)\sqrt{-1-i} + (4-i)\sqrt{-1+i} + 2(-1-i)^{3/2}\sqrt{-1+i} \right)$$

$$\left. \text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)}, \text{ArcSin}\left[\frac{\sqrt{\frac{\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left(-1-2i\right)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2}}{\sqrt{-1+i}\left(-1+2i\right)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2}}\right], 4-2\sqrt{2}\right] \right)$$

$$\text{Sec}\left[\frac{x}{2}\right]^2 \text{Tan}\left[\frac{x}{2}\right] \sqrt{\frac{\left(2i+\sqrt{-1-i}-\sqrt{-1+i}\right)\left(\left(1-2i\right)+2\sqrt{-1-i}-\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\left(-2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left(-1+2i\right)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2}}\left(\left(-1+2i\right)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

$$\sqrt{-\frac{\left(1-i\right)\left(-1+2i\right)+2\sqrt{-1-i}\left(1-\left(2+4i\right)\text{Tan}\left[\frac{x}{2}\right]^2+\text{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(-1+2i\right)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2}} \left/ \left(\sqrt{-1+i}\left(-12+4i\right)+\left(7+8i\right)\sqrt{-1-i}\right)\right.$$

$$\left. \left(\left(2+2i\right)-\left(2-i\right)\sqrt{-1+i}\right)\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2 \sqrt{\frac{1-4\text{Tan}\left[\frac{x}{2}\right]^2+22\text{Tan}\left[\frac{x}{2}\right]^4-4\text{Tan}\left[\frac{x}{2}\right]^6+\text{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}}\right) + \right.$$

$$\left. \left( \left(2+6i\right) - \frac{8}{\sqrt{-1-i}} - 5\sqrt{-1+i} + \left(2+4i\right)\sqrt{2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left(-1-2i\right)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2}}{\sqrt{-1+i}\left(-1+2i\right)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2}}\right], 4-2\sqrt{2}\right] + \right.$$

$$\left. \left(\left(-4-4i\right)-\left(3-5i\right)\sqrt{-1-i}+\left(5-3i\right)\sqrt{-1+i}+4\left(-1-i\right)^{3/2}\sqrt{-1+i}\right) \right.$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left(-1+i\right)+\sqrt{-1-i}}{\left(-1-i\right)+\sqrt{-1+i}\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)}, \text{ArcSin}\left[\frac{\sqrt{\frac{\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left(-1-2i\right)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2}}{\sqrt{-1+i}\left(-1+2i\right)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2}}\right], 4-2\sqrt{2}\right] +$$

$$\left(\left(-4-4i\right)-\left(1-4i\right)\sqrt{-1-i}+\left(4-i\right)\sqrt{-1+i}+2\left(-1-i\right)^{3/2}\sqrt{-1+i}\right)$$

$$\left. \text{EllipticPi} \left[ \frac{2\sqrt{-1+i} (i + \sqrt{-1-i})}{(-i + \sqrt{-1+i}) (2i + \sqrt{-1-i} + \sqrt{-1+i})}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(2i + \sqrt{-1-i} + \sqrt{-1+i}) \left( (-1-2i) + 2\sqrt{-1+i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}{\sqrt{-1+i} \left( (-1+2i) + 2\sqrt{-1-i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}}}{\sqrt{2}} \right], 4 - 2\sqrt{2} \right] \right)$$

$$\text{Sec} \left[ \frac{x}{2} \right]^2 \text{Tan} \left[ \frac{x}{2} \right] \sqrt{\frac{(2i + \sqrt{-1-i} - \sqrt{-1+i}) \left( (1-2i) + 2\sqrt{-1-i} - \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}{(-2i + \sqrt{-1-i} + \sqrt{-1+i}) \left( (-1+2i) + 2\sqrt{-1-i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}} \left( (-1+2i) + 2\sqrt{-1-i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)^2$$

$$\sqrt{\frac{(1-i) \left( (-1+2i) + 2\sqrt{-1-i} \right) \left( 1 - (2+4i) \text{Tan} \left[ \frac{x}{2} \right]^2 + \text{Tan} \left[ \frac{x}{2} \right]^4 \right)}{\left( (-1+2i) + 2\sqrt{-1-i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)^2}} \left/ \left( \sqrt{-1+i} \left( (-12+4i) + (7+8i) \sqrt{-1-i} \right) \right) \right.$$

$$\left. \left( (2+2i) - (2-i) \sqrt{-1+i} \right) \left( 1 + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)^3 \sqrt{\frac{1 - 4 \text{Tan} \left[ \frac{x}{2} \right]^2 + 22 \text{Tan} \left[ \frac{x}{2} \right]^4 - 4 \text{Tan} \left[ \frac{x}{2} \right]^6 + \text{Tan} \left[ \frac{x}{2} \right]^8}{\left( 1 + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)^4}} \right) -$$

$$2 \left( \left( (2+6i) - \frac{8}{\sqrt{-1-i}} - 5\sqrt{-1+i} + (2+4i)\sqrt{2} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(2i + \sqrt{-1-i} + \sqrt{-1+i}) \left( (-1-2i) + 2\sqrt{-1+i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}{\sqrt{-1+i} \left( (-1+2i) + 2\sqrt{-1-i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}}}{\sqrt{2}} \right], 4 - 2\sqrt{2} \right] + \right.$$

$$\left. \left( (-4-4i) - (3-5i) \sqrt{-1-i} + (5-3i) \sqrt{-1+i} + 4(-1-i)^{3/2} \sqrt{-1+i} \right) \right.$$

$$\text{EllipticPi} \left[ \frac{2\sqrt{-1+i} \left( (-1+i) + \sqrt{-1-i} \right)}{\left( (-1-i) + \sqrt{-1+i} \right) \left( 2i + \sqrt{-1-i} + \sqrt{-1+i} \right)}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(2i + \sqrt{-1-i} + \sqrt{-1+i}) \left( (-1-2i) + 2\sqrt{-1+i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}{\sqrt{-1+i} \left( (-1+2i) + 2\sqrt{-1-i} + \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}}}{\sqrt{2}} \right], 4 - 2\sqrt{2} \right] +$$

$$\left( (-4-4i) - (1-4i) \sqrt{-1-i} + (4-i) \sqrt{-1+i} + 2(-1-i)^{3/2} \sqrt{-1+i} \right)$$

$$\left. \text{EllipticPi} \left[ \frac{2\sqrt{-1+i} (i + \sqrt{-1-i})}{(-i + \sqrt{-1+i}) (2i + \sqrt{-1-i} + \sqrt{-1+i})}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(2i + \sqrt{-1-i} + \sqrt{-1+i}) ((-1-2i) + 2\sqrt{-1+i} + \tan[\frac{x}{2}]^2)}{\sqrt{-1+i} ((-1+2i) + 2\sqrt{-1-i} + \tan[\frac{x}{2}]^2)}}}{\sqrt{2}} \right], 4 - 2\sqrt{2} \right] \right)$$

$$\left( (-1 + 2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2 \right)^2 \sqrt{-\frac{(1-i) ((-1+2i) + 2\sqrt{-1-i}) (1 - (2+4i) \tan\left[\frac{x}{2}\right]^2 + \tan\left[\frac{x}{2}\right]^4)}{(((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2))^2}}$$

$$\left( -\frac{(2i + \sqrt{-1-i} - \sqrt{-1+i}) \text{Sec}\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] ((1-2i) + 2\sqrt{-1-i} - \tan\left[\frac{x}{2}\right]^2)}{(-2i + \sqrt{-1-i} + \sqrt{-1+i}) (((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2))^2} - \right.$$

$$\left. \frac{(2i + \sqrt{-1-i} - \sqrt{-1+i}) \text{Sec}\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]}{(-2i + \sqrt{-1-i} + \sqrt{-1+i}) (((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2))} \right) /$$

$$\left( \sqrt{-1+i} ((-12+4i) + (7+8i)\sqrt{-1-i}) ((2+2i) - (2-i)\sqrt{-1+i}) (1 + \tan\left[\frac{x}{2}\right]^2)^2 \right.$$

$$\left. \sqrt{\frac{(2i + \sqrt{-1-i} - \sqrt{-1+i}) ((1-2i) + 2\sqrt{-1-i} - \tan\left[\frac{x}{2}\right]^2)}{(-2i + \sqrt{-1-i} + \sqrt{-1+i}) (((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2))}} \sqrt{\frac{1 - 4 \tan\left[\frac{x}{2}\right]^2 + 22 \tan\left[\frac{x}{2}\right]^4 - 4 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{(1 + \tan\left[\frac{x}{2}\right]^2)^4}} \right) -$$

$$2 \left( (2+6i) - \frac{8}{\sqrt{-1-i}} - 5\sqrt{-1+i} + (2+4i)\sqrt{2} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(2i + \sqrt{-1-i} + \sqrt{-1+i}) ((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2)}{\sqrt{-1+i} ((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2)}}}{\sqrt{2}} \right], 4 - 2\sqrt{2} \right] +$$

$$\left( (-4-4i) - (3-5i)\sqrt{-1-i} + (5-3i)\sqrt{-1+i} + 4(-1-i)^{3/2}\sqrt{-1+i} \right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left((-1+i)+\sqrt{-1-i}\right)}{\left((-1-i)+\sqrt{-1+i}\right)\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)}, \text{ArcSin}\left[\frac{\sqrt{\frac{\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2i)+2\sqrt{-1+i}+\tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2i)+2\sqrt{-1-i}+\tan\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right], 4-2\sqrt{2}\right] +$$

$$\left. \left( (-4-4i) - (1-4i)\sqrt{-1-i} + (4-i)\sqrt{-1+i} + 2(-1-i)^{3/2}\sqrt{-1+i} \right) \right.$$

$$\left. \text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)}, \text{ArcSin}\left[\frac{\sqrt{\frac{\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2i)+2\sqrt{-1+i}+\tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2i)+2\sqrt{-1-i}+\tan\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right], 4-2\sqrt{2}\right] \right)$$

$$\sqrt{\frac{\left(2i+\sqrt{-1-i}-\sqrt{-1+i}\right)\left(\left(1-2i\right)+2\sqrt{-1-i}-\tan\left[\frac{x}{2}\right]^2\right)}{\left(-2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left(\left(-1+2i\right)+2\sqrt{-1-i}+\tan\left[\frac{x}{2}\right]^2\right)}} \left(\left(-1+2i\right)+2\sqrt{-1-i}+\tan\left[\frac{x}{2}\right]^2\right)^2$$

$$\left( -\frac{\left(1-i\right)\left(\left(-1+2i\right)+2\sqrt{-1-i}\right)\left(\left(-2-4i\right)\sec\left[\frac{x}{2}\right]^2\tan\left[\frac{x}{2}\right]+2\sec\left[\frac{x}{2}\right]^2\tan\left[\frac{x}{2}\right]^3\right)}{\left(\left(-1+2i\right)+2\sqrt{-1-i}+\tan\left[\frac{x}{2}\right]^2\right)^2} + \right.$$

$$\left. \frac{\left(2-2i\right)\left(\left(-1+2i\right)+2\sqrt{-1-i}\right)\sec\left[\frac{x}{2}\right]^2\tan\left[\frac{x}{2}\right]\left(1-\left(2+4i\right)\tan\left[\frac{x}{2}\right]^2+\tan\left[\frac{x}{2}\right]^4\right)}{\left(\left(-1+2i\right)+2\sqrt{-1-i}+\tan\left[\frac{x}{2}\right]^2\right)^3} \right) /$$

$$\left( \sqrt{-1+i}\left(\left(-12+4i\right)+\left(7+8i\right)\sqrt{-1-i}\right)\left(\left(2+2i\right)-\left(2-i\right)\sqrt{-1+i}\right)\left(1+\tan\left[\frac{x}{2}\right]^2\right)^2 \right.$$

$$\left. \sqrt{-\frac{\left(1-i\right)\left(\left(-1+2i\right)+2\sqrt{-1-i}\right)\left(1-\left(2+4i\right)\tan\left[\frac{x}{2}\right]^2+\tan\left[\frac{x}{2}\right]^4\right)}{\left(\left(-1+2i\right)+2\sqrt{-1-i}+\tan\left[\frac{x}{2}\right]^2\right)^2}} \sqrt{\frac{1-4\tan\left[\frac{x}{2}\right]^2+22\tan\left[\frac{x}{2}\right]^4-4\tan\left[\frac{x}{2}\right]^6+\tan\left[\frac{x}{2}\right]^8}{\left(1+\tan\left[\frac{x}{2}\right]^2\right)^4}} \right) +$$

$$\begin{aligned}
& 2 \left( \left( (2+6i) - \frac{8}{\sqrt{-1-i}} - 5\sqrt{-1+i} + (2+4i)\sqrt{2} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{(2i+\sqrt{-1-i}+\sqrt{-1+i})((-1-2i)+2\sqrt{-1+i}+\text{Tan}[\frac{x}{2}]^2)}{\sqrt{-1+i}((-1+2i)+2\sqrt{-1-i}+\text{Tan}[\frac{x}{2}]^2)}}}{\sqrt{2}} \right], 4-2\sqrt{2} \right] + \right. \\
& \left( (-4-4i) - (3-5i)\sqrt{-1-i} + (5-3i)\sqrt{-1+i} + 4(-1-i)^{3/2}\sqrt{-1+i} \right) \\
& \text{EllipticPi} \left[ \frac{2\sqrt{-1+i}((-1+i)+\sqrt{-1-i})}{((-1-i)+\sqrt{-1+i})(2i+\sqrt{-1-i}+\sqrt{-1+i})}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(2i+\sqrt{-1-i}+\sqrt{-1+i})((-1-2i)+2\sqrt{-1+i}+\text{Tan}[\frac{x}{2}]^2)}{\sqrt{-1+i}((-1+2i)+2\sqrt{-1-i}+\text{Tan}[\frac{x}{2}]^2)}}}{\sqrt{2}} \right], 4-2\sqrt{2} \right] + \\
& \left( (-4-4i) - (1-4i)\sqrt{-1-i} + (4-i)\sqrt{-1+i} + 2(-1-i)^{3/2}\sqrt{-1+i} \right) \\
& \left. \text{EllipticPi} \left[ \frac{2\sqrt{-1+i}(i+\sqrt{-1-i})}{(-i+\sqrt{-1+i})(2i+\sqrt{-1-i}+\sqrt{-1+i})}, \text{ArcSin} \left[ \frac{\sqrt{\frac{(2i+\sqrt{-1-i}+\sqrt{-1+i})((-1-2i)+2\sqrt{-1+i}+\text{Tan}[\frac{x}{2}]^2)}{\sqrt{-1+i}((-1+2i)+2\sqrt{-1-i}+\text{Tan}[\frac{x}{2}]^2)}}}{\sqrt{2}} \right], 4-2\sqrt{2} \right] \right) \\
& \sqrt{\frac{(2i+\sqrt{-1-i}-\sqrt{-1+i})((1-2i)+2\sqrt{-1-i}-\text{Tan}[\frac{x}{2}]^2)}{(-2i+\sqrt{-1-i}+\sqrt{-1+i})((-1+2i)+2\sqrt{-1-i}+\text{Tan}[\frac{x}{2}]^2)}}{((-1+2i)+2\sqrt{-1-i}+\text{Tan}[\frac{x}{2}]^2)^2}} \\
& \sqrt{\frac{(1-i)((-1+2i)+2\sqrt{-1-i})(1-(2+4i)\text{Tan}[\frac{x}{2}]^2+\text{Tan}[\frac{x}{2}]^4)}{((-1+2i)+2\sqrt{-1-i}+\text{Tan}[\frac{x}{2}]^2)^2}} \\
& \left( \frac{-4\text{Sec}[\frac{x}{2}]^2\text{Tan}[\frac{x}{2}] + 44\text{Sec}[\frac{x}{2}]^2\text{Tan}[\frac{x}{2}]^3 - 12\text{Sec}[\frac{x}{2}]^2\text{Tan}[\frac{x}{2}]^5 + 4\text{Sec}[\frac{x}{2}]^2\text{Tan}[\frac{x}{2}]^7}{(1+\text{Tan}[\frac{x}{2}]^2)^4} - \right. \\
& \left. \frac{4\text{Sec}[\frac{x}{2}]^2\text{Tan}[\frac{x}{2}](1-4\text{Tan}[\frac{x}{2}]^2+22\text{Tan}[\frac{x}{2}]^4-4\text{Tan}[\frac{x}{2}]^6+\text{Tan}[\frac{x}{2}]^8)}{(1+\text{Tan}[\frac{x}{2}]^2)^5} \right) \Bigg/ \left( \sqrt{-1+i}((-12+4i)+(7+8i)\sqrt{-1-i}) \right)
\end{aligned}$$



$$\begin{aligned}
 & \left( (2+2i) - (2-i) \sqrt{-1+i} \right) \left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^2 \left( \frac{1 - 4 \tan\left[\frac{x}{2}\right]^2 + 22 \tan\left[\frac{x}{2}\right]^4 - 4 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{\left(1 + \tan\left[\frac{x}{2}\right]^2\right)^4} \right)^{3/2} \right) - \\
 & \left( 4 \sqrt{\frac{\left(2i + \sqrt{-1-i} - \sqrt{-1+i}\right) \left(\left(1-2i\right) + 2\sqrt{-1-i} - \tan\left[\frac{x}{2}\right]^2\right)}{\left(-2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)} \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)^2} \right. \\
 & \sqrt{-\frac{\left(1-i\right) \left(\left(-1+2i\right) + 2\sqrt{-1-i}\right) \left(1 - \left(2+4i\right) \tan\left[\frac{x}{2}\right]^2 + \tan\left[\frac{x}{2}\right]^4\right)}{\left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)^2}} \\
 & \left. \left( \left( \left( \left(2+6i\right) - \frac{8}{\sqrt{-1-i}} - 5\sqrt{-1+i} + \left(2+4i\right) \sqrt{2} \right) \left( \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]}{\sqrt{-1+i} \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)} - \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] \left(\left(-1-2i\right) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i} \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)^2} \right) \right) \right) / \\
 & \left( 2\sqrt{2} \sqrt{\frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left(\left(-1-2i\right) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i} \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} \right. \\
 & \sqrt{1 - \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left(\left(-1-2i\right) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{2\sqrt{-1+i} \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} \\
 & \left. \sqrt{1 - \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left(4 - 2\sqrt{2}\right) \left(\left(-1-2i\right) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{2\sqrt{-1+i} \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} \right) + \\
 & \left( \left( \left( \left(-4-4i\right) - \left(3-5i\right) \sqrt{-1-i} + \left(5-3i\right) \sqrt{-1+i} + 4 \left(-1-i\right)^{3/2} \sqrt{-1+i} \right) \left( \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]}{\sqrt{-1+i} \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)} - \right. \right. \right. \\
 & \left. \left. \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] \left(\left(-1-2i\right) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i} \left(\left(-1+2i\right) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)^2} \right) \right) \right) / \left( 2\sqrt{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \sqrt{\frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i} \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} \left(1 - \frac{\left((-1+i) + \sqrt{-1-i}\right) \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{\left((-1-i) + \sqrt{-1+i}\right) \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}\right) \\
 & \sqrt{1 - \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{2\sqrt{-1+i} \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} \\
 & \sqrt{1 - \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left(4 - 2\sqrt{2}\right) \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{2\sqrt{-1+i} \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} + \\
 & \left( \left( (-4-4i) - (1-4i)\sqrt{-1-i} + (4-i)\sqrt{-1+i} + 2(-1-i)^{3/2}\sqrt{-1+i} \right) \left( \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \operatorname{Sec}\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]}{\sqrt{-1+i} \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)} - \right. \right. \\
 & \left. \left. \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \operatorname{Sec}\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i} \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)^2} \right) \right) / \\
 & \left( 2\sqrt{2} \sqrt{\frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i} \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} \left( 1 - \frac{\left(i + \sqrt{-1-i}\right) \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{\left(-i + \sqrt{-1+i}\right) \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)} \right) \right) \\
 & \sqrt{1 - \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{2\sqrt{-1+i} \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} \\
 & \sqrt{1 - \frac{\left(2i + \sqrt{-1-i} + \sqrt{-1+i}\right) \left(4 - 2\sqrt{2}\right) \left((-1-2i) + 2\sqrt{-1+i} + \tan\left[\frac{x}{2}\right]^2\right)}{2\sqrt{-1+i} \left((-1+2i) + 2\sqrt{-1-i} + \tan\left[\frac{x}{2}\right]^2\right)}} \right) / \\
 & \left( \sqrt{-1+i} \left((-12+4i) + (7+8i)\sqrt{-1-i}\right) \left((2+2i) - (2-i)\sqrt{-1+i}\right) \left(1 + \tan\left[\frac{x}{2}\right]^2\right)^2 \right)
 \end{aligned}$$

$$\sqrt{\frac{1 - 4 \tan\left[\frac{x}{2}\right]^2 + 22 \tan\left[\frac{x}{2}\right]^4 - 4 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{\left(1 + \tan\left[\frac{x}{2}\right]^2\right)^4}}$$

Problem 7: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\tan[x]}{\sqrt{1 + \sec[x]^3}} dx$$

Optimal (type 3, 15 leaves, 4 steps):

$$-\frac{2}{3} \operatorname{ArcTanh}\left[\sqrt{1 + \sec[x]^3}\right]$$

Result (type 4, 3292 leaves):

$$-\left( i \cos[x]^2 \left( \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{3} \sqrt{\frac{i \cos[x] \sec\left[\frac{x}{2}\right]^2}{-3i + \sqrt{3}}}\right], \frac{3i - \sqrt{3}}{3i + \sqrt{3}} \right] - \operatorname{EllipticPi}\left[ \frac{1}{6} (3 + i\sqrt{3}), i \operatorname{ArcSinh}\left[ \sqrt{3} \sqrt{\frac{i \cos[x] \sec\left[\frac{x}{2}\right]^2}{-3i + \sqrt{3}}}\right], \frac{3i - \sqrt{3}}{3i + \sqrt{3}} \right] \right) \sec\left[\frac{x}{2}\right]^4 \sqrt{(4 + 3 \cos[x] + \cos[3x]) \sec[x]^3} \right. \\ \left. - \frac{\sqrt{\frac{4}{3 \cos[x] + \cos[3x]} + \frac{3 \cos[x]}{3 \cos[x] + \cos[3x]} + \frac{\cos[3x]}{3 \cos[x] + \cos[3x]}} \sec\left[\frac{x}{2}\right] \sin\left[\frac{3x}{2}\right]}{2(3 - 2 \cos[x] + \cos[2x])} + \frac{\sqrt{\frac{4}{3 \cos[x] + \cos[3x]} + \frac{3 \cos[x]}{3 \cos[x] + \cos[3x]} + \frac{\cos[3x]}{3 \cos[x] + \cos[3x]}} \sec\left[\frac{x}{2}\right] \sin\left[\frac{5x}{2}\right]}{2(3 - 2 \cos[x] + \cos[2x])} \right. \\ \left. + \frac{\sqrt{\frac{4}{3 \cos[x] + \cos[3x]} + \frac{3 \cos[x]}{3 \cos[x] + \cos[3x]} + \frac{\cos[3x]}{3 \cos[x] + \cos[3x]}} \tan\left[\frac{x}{2}\right]}{3 - 2 \cos[x] + \cos[2x]} \sqrt{\frac{\sqrt{3} - 3i \tan\left[\frac{x}{2}\right]^2}{-3i + \sqrt{3}}} \sqrt{\frac{\sqrt{3} + 3i \tan\left[\frac{x}{2}\right]^2}{3i + \sqrt{3}}} \right)$$

$$\begin{aligned}
& \left( \sqrt{3} \sqrt{\frac{\cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3-i\sqrt{3}}} \left(1+3 \operatorname{Tan}\left[\frac{x}{2}\right]^4\right) \left( 2 i \sqrt{3} \cos[x]^2 \left( \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3+i\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right] - \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{1}{6}(3+i\sqrt{3}), i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3+i\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right] \right) \operatorname{Sec}\left[\frac{x}{2}\right]^6 \sqrt{(4+3 \cos[x]+\cos[3 x]) \operatorname{Sec}[x]^3} \right. \\
& \left. \operatorname{Tan}\left[\frac{x}{2}\right]^3 \sqrt{\frac{\sqrt{3}-3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{-3+i\sqrt{3}}} \sqrt{\frac{\sqrt{3}+3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{3 i+\sqrt{3}}} \right) / \left( \sqrt{\frac{\cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3-i\sqrt{3}}} \left(1+3 \operatorname{Tan}\left[\frac{x}{2}\right]^4\right)^2 \right) + \\
& \left( \sqrt{3} \cos[x]^2 \left( \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3+i\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right] - \operatorname{EllipticPi}\left[\frac{1}{6}(3+i\sqrt{3}), \right. \right. \\
& \left. \left. i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3+i\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right] \right) \operatorname{Sec}\left[\frac{x}{2}\right]^6 \sqrt{(4+3 \cos[x]+\cos[3 x]) \operatorname{Sec}[x]^3} \right. \\
& \left. \operatorname{Tan}\left[\frac{x}{2}\right] \sqrt{\frac{\sqrt{3}-3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{-3+i\sqrt{3}}} \right) / \left( 2(3 i+\sqrt{3}) \sqrt{\frac{\cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3-i\sqrt{3}}} \sqrt{\frac{\sqrt{3}+3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{3 i+\sqrt{3}}} \left(1+3 \operatorname{Tan}\left[\frac{x}{2}\right]^4\right) \right) - \\
& \left( \sqrt{3} \cos[x]^2 \left( \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3+i\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right] - \operatorname{EllipticPi}\left[\frac{1}{6}(3+i\sqrt{3}), \right. \right. \\
& \left. \left. i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3+i\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right] \right) \operatorname{Sec}\left[\frac{x}{2}\right]^6 \sqrt{(4+3 \cos[x]+\cos[3 x]) \operatorname{Sec}[x]^3} \operatorname{Tan}\left[\frac{x}{2}\right] \sqrt{\frac{\sqrt{3}+3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{3 i+\sqrt{3}}} \right) /
\end{aligned}$$

$$\begin{aligned}
 & \left( 2 (-3 i + \sqrt{3}) \sqrt{\frac{\cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 - i \sqrt{3}}} \sqrt{\frac{\sqrt{3} - 3 i \tan\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}} \left(1 + 3 \tan\left[\frac{x}{2}\right]^4\right) \right) + \left( 2 i \cos[x] \left( \operatorname{EllipticF}\left[ \right. \right. \right. \\
 & \quad \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}}\right], \frac{3 i - \sqrt{3}}{3 i + \sqrt{3}}\right] - \operatorname{EllipticPi}\left[\frac{1}{6} (3 + i \sqrt{3})\right], i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}}\right], \frac{3 i - \sqrt{3}}{3 i + \sqrt{3}}\right] \right) \\
 & \operatorname{Sec}\left[\frac{x}{2}\right]^4 \sqrt{(4 + 3 \cos[x] + \cos[3x]) \operatorname{Sec}[x]^3 \sin[x]} \sqrt{\frac{\sqrt{3} - 3 i \tan\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}} \sqrt{\frac{\sqrt{3} + 3 i \tan\left[\frac{x}{2}\right]^2}{3 i + \sqrt{3}}} \Big/ \\
 & \left( \sqrt{3} \sqrt{\frac{\cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 - i \sqrt{3}}} \left(1 + 3 \tan\left[\frac{x}{2}\right]^4\right) \right) - \left( 2 i \cos[x]^2 \left( \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}}\right], \frac{3 i - \sqrt{3}}{3 i + \sqrt{3}}\right] - \right. \right. \\
 & \quad \left. \left. \operatorname{EllipticPi}\left[\frac{1}{6} (3 + i \sqrt{3})\right], i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}}\right], \frac{3 i - \sqrt{3}}{3 i + \sqrt{3}}\right] \right) \operatorname{Sec}\left[\frac{x}{2}\right]^4 \sqrt{(4 + 3 \cos[x] + \cos[3x]) \operatorname{Sec}[x]^3} \\
 & \tan\left[\frac{x}{2}\right] \sqrt{\frac{\sqrt{3} - 3 i \tan\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}} \sqrt{\frac{\sqrt{3} + 3 i \tan\left[\frac{x}{2}\right]^2}{3 i + \sqrt{3}}} \Big/ \left( \sqrt{3} \sqrt{\frac{\cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 - i \sqrt{3}}} \left(1 + 3 \tan\left[\frac{x}{2}\right]^4\right) \right) + \\
 & \left( i \cos[x]^2 \left( \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}}\right], \frac{3 i - \sqrt{3}}{3 i + \sqrt{3}}\right] - \operatorname{EllipticPi}\left[\frac{1}{6} (3 + i \sqrt{3})\right], \right. \right. \\
 & \quad \left. \left. i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}}\right], \frac{3 i - \sqrt{3}}{3 i + \sqrt{3}}\right] \right) \operatorname{Sec}\left[\frac{x}{2}\right]^4 \sqrt{(4 + 3 \cos[x] + \cos[3x]) \operatorname{Sec}[x]^3} \left( -\frac{\operatorname{Sec}\left[\frac{x}{2}\right]^2 \sin[x]}{-3 - i \sqrt{3}} + \right. \\
 & \quad \left. \frac{\cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]}{-3 - i \sqrt{3}} \right) \sqrt{\frac{\sqrt{3} - 3 i \tan\left[\frac{x}{2}\right]^2}{-3 i + \sqrt{3}}} \sqrt{\frac{\sqrt{3} + 3 i \tan\left[\frac{x}{2}\right]^2}{3 i + \sqrt{3}}} \Big/ \left( 2 \sqrt{3} \left( \frac{\cos[x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 - i \sqrt{3}} \right)^{3/2} \left(1 + 3 \tan\left[\frac{x}{2}\right]^4\right) \right) -
 \end{aligned}$$

$$\left( i \cos [x]^2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 \sqrt{(4+3 \cos [x]+\cos [3 x]) \operatorname{Sec}[x]^3} \sqrt{\frac{\sqrt{3}-3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}} \sqrt{\frac{\sqrt{3}+3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{3 i+\sqrt{3}}} \right.$$

$$\left. \frac{i \sqrt{3}\left(-\frac{i \operatorname{Sec}\left[\frac{x}{2}\right]^2 \sin [x]}{-3 i+\sqrt{3}}+\frac{i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2 \operatorname{Tan}\left[\frac{x}{2}\right]}{-3 i+\sqrt{3}}\right)}{2 \sqrt{\frac{i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}} \sqrt{1+\frac{3 i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}} \sqrt{1+\frac{3 i(3 i-\sqrt{3}) \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{(-3 i+\sqrt{3})(3 i+\sqrt{3})}}}-\left(i \sqrt{3}\left(-\frac{i \operatorname{Sec}\left[\frac{x}{2}\right]^2 \sin [x]}{-3 i+\sqrt{3}}+\frac{i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2 \operatorname{Tan}\left[\frac{x}{2}\right]}{-3 i+\sqrt{3}}\right)\right) /$$

$$\left( 2 \sqrt{\frac{i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}} \sqrt{1+\frac{3 i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}} \left(1+\frac{i(3+i \sqrt{3}) \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{2(-3 i+\sqrt{3})}\right) \sqrt{1+\frac{3 i(3 i-\sqrt{3}) \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{(-3 i+\sqrt{3})(3 i+\sqrt{3})}} \right) /$$

$$\left( \sqrt{3} \sqrt{\frac{\cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3-i \sqrt{3}}}\left(1+3 \operatorname{Tan}\left[\frac{x}{2}\right]^4\right)-\left(i \cos [x]^2\left(\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right]-\right. \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{1}{6}(3+i \sqrt{3}), i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right]\right) \operatorname{Sec}\left[\frac{x}{2}\right]^4 \sqrt{\frac{\sqrt{3}-3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}$$

$$\left. \sqrt{\frac{\sqrt{3}+3 i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{3 i+\sqrt{3}}}\left(\operatorname{Sec}[x]^3(-3 \sin [x]-3 \sin [3 x])+3(4+3 \cos [x]+\cos [3 x]) \operatorname{Sec}[x]^3 \operatorname{Tan}[x]\right) / \right)$$

$$\left( 2 \sqrt{3} \sqrt{\frac{\cos[x] \sec\left[\frac{x}{2}\right]^2}{-3 - i \sqrt{3}}} \sqrt{(4 + 3 \cos[x] + \cos[3x]) \sec[x]^3} \left(1 + 3 \tan\left[\frac{x}{2}\right]^4\right) \right) \Bigg) \Bigg) \Bigg)$$

**Problem 8: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{2 + 2 \tan[x] + \tan[x]^2} \, dx$$

Optimal (type 3, 137 leaves, 9 steps):

$$\text{ArcSinh}[1 + \tan[x]] - \sqrt{\frac{1}{2} (1 + \sqrt{5})} \text{ArcTan}\left[\frac{2\sqrt{5} - (5 + \sqrt{5}) \tan[x]}{\sqrt{10(1 + \sqrt{5})} \sqrt{2 + 2 \tan[x] + \tan[x]^2}}\right] -$$

$$\sqrt{\frac{1}{2} (-1 + \sqrt{5})} \text{ArcTanh}\left[\frac{2\sqrt{5} + (5 - \sqrt{5}) \tan[x]}{\sqrt{10(-1 + \sqrt{5})} \sqrt{2 + 2 \tan[x] + \tan[x]^2}}\right]$$

Result (type 4, 7376 leaves):

$$-\frac{1}{(1 + \cos[x]) \sqrt{\frac{3 + \cos[2x] + 2 \sin[2x]}{(1 + \cos[x])^2}}} 4 \cos[x]$$

$$\left( \left( \left( \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2\right] - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4\right]\right) \left(-\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right] + \tan\left[\frac{x}{2}\right]\right)\right)}\right]} \right) \right) \right) /$$

$$\left( \left( \left( \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right] - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4\right]\right) \left(-\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2\right] + \tan\left[\frac{x}{2}\right]\right) \right) \right) \right) /$$

$$\left( \left( \left( \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2\right] - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 3\right]\right) \left(\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right] - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4\right]\right) \right) \right) /$$

$$\left( \left( \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 3\right]\right) \left(\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2\right] - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4\right]\right) \right) \right) \left(1 - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right] - \text{EllipticPi}\left[\left(\left(-1 + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2\right]\right) \left(-\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4\right]\right)\right) / \left(\left(-1 + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right]\right) \left(-\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4\right]\right)\right), \right.$$

$$\left. \text{ArcSin}\left[\sqrt{\left(\left(\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2\right] - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4\right]\right) \left(-\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right] + \tan\left[\frac{x}{2}\right]\right)\right)}\right] \right) /$$

$$\begin{aligned}
& \left( \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \Bigg], \\
& - \left( \left( \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 3] \right) \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] - \right. \right. \right. \\
& \quad \left. \left. \left. \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \right) / \left( \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 3] \right) \right. \right. \\
& \quad \left. \left. \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \right) \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \right. \\
& \quad \left. \left. \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] \right) \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \\
& \sqrt{\left( \left( \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) /} \\
& \left( \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \right. \\
& \quad \left. \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] + \text{Tan}\left[\frac{x}{2}\right] \right)^2 \\
& \sqrt{\left( \left( \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 3] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) /} \\
& \left( \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 3] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \\
& \sqrt{\left( \left( \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) /} \\
& \left( \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \Bigg) /} \\
& \left( \left( -1 + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] \right) \left( 1 - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] \right) \right. \\
& \quad \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] \right) \\
& \quad \left. \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \right. \\
& \quad \left. \sqrt{1 + 2 \text{Tan}\left[\frac{x}{2}\right] - 2 \text{Tan}\left[\frac{x}{2}\right]^3 + \text{Tan}\left[\frac{x}{2}\right]^4} \right) - \\
& \left( (2 - i) \left( \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left( \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \right. \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) / \left( \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] + \right. \right. \\
& \quad \left. \left. \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) \Bigg], - \left( \left( \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 3] \right) \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] - \right. \right. \right. \\
& \quad \left. \left. \left. \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \right) \right) / \left( \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 3] \right) \right. \right. \\
& \quad \left. \left. \left( \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \right) \right) \left( i - \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] \right) - \\
& \text{EllipticPi}\left[ \left( \left( -i + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \right) \right) / \\
& \quad \left( \left( -i + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 1] \right) \left( -\text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 2] + \text{Root}[1 + 2 \#1 - 2 \#1^3 + \#1^4 \&, 4] \right) \right) \Bigg],
\end{aligned}$$









$$\begin{aligned}
& \left( \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 4\right] \right) \\
& \left( \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 1\right] - \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 4\right] \right) \\
& \sqrt{\left( \left( \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 4\right] \right) \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 1\right] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) /} \\
& \left( \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 1\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 4\right] \right) \right. \\
& \left. \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] + \text{Tan}\left[\frac{x}{2}\right] \right)^2 \\
& \sqrt{\left( \left( \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 1\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] \right) \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 3\right] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) /} \\
& \left( \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 1\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 3\right] \right) \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \\
& \sqrt{\left( \left( \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 1\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] \right) \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 4\right] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) /} \\
& \left( \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 1\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 4\right] \right) \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] + \text{Tan}\left[\frac{x}{2}\right] \right) \right) \right) /} \\
& \left( \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 1\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] \right) \left( -\text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 2\right] + \text{Root}\left[1 + 2 \#1 - 2 \#1^3 + \#1^4, 4\right] \right) \right. \\
& \left. \left. \sqrt{1 + 2 \text{Tan}\left[\frac{x}{2}\right] - 2 \text{Tan}\left[\frac{x}{2}\right]^3 + \text{Tan}\left[\frac{x}{2}\right]^4} \right) \right) \sqrt{2 + 2 \text{Tan}[x] + \text{Tan}[x]^2}
\end{aligned}$$

**Problem 9: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{ArcTan}\left[\sqrt{-1 + \text{Sec}[x]}\right] \text{Sin}[x] \, dx$$

Optimal (type 3, 41 leaves, 7 steps):

$$\frac{1}{2} \text{ArcTan}\left[\sqrt{-1 + \text{Sec}[x]}\right] - \text{ArcTan}\left[\sqrt{-1 + \text{Sec}[x]}\right] \text{Cos}[x] + \frac{1}{2} \text{Cos}[x] \sqrt{-1 + \text{Sec}[x]}$$

Result (type 4, 285 leaves):

$$\begin{aligned}
& -\text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right] \text{Cos}[x] + \frac{1}{2} \text{Cos}[x] \sqrt{-1+\text{Sec}[x]} - \frac{1}{2} (-3-2\sqrt{2}) \text{Cos}\left[\frac{x}{4}\right]^2 \left(1-\sqrt{2} + (-2+\sqrt{2}) \text{Cos}\left[\frac{x}{2}\right]\right) \\
& \text{Cot}\left[\frac{x}{4}\right] \left( \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \sqrt{\left(7-5\sqrt{2} + (10-7\sqrt{2}) \text{Cos}\left[\frac{x}{2}\right]\right) \text{Sec}\left[\frac{x}{4}\right]^2} \sqrt{\left(-1-\sqrt{2} + (2+\sqrt{2}) \text{Cos}\left[\frac{x}{2}\right]\right) \text{Sec}\left[\frac{x}{4}\right]^2} \\
& \sqrt{-1+\text{Sec}[x]} \text{Sec}[x] \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{x}{4}\right]^2} \sqrt{1+(-3+2\sqrt{2}) \text{Tan}\left[\frac{x}{4}\right]^2}
\end{aligned}$$

**Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{ArcTan}\left[x + \sqrt{1-x^2}\right] dx$$

Optimal (type 3, 141 leaves, ? steps):

$$\begin{aligned}
& -\frac{\text{ArcSin}[x]}{2} + \frac{1}{4} \sqrt{3} \text{ArcTan}\left[\frac{-1+\sqrt{3}x}{\sqrt{1-x^2}}\right] + \frac{1}{4} \sqrt{3} \text{ArcTan}\left[\frac{1+\sqrt{3}x}{\sqrt{1-x^2}}\right] - \\
& \frac{1}{4} \sqrt{3} \text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right] + x \text{ArcTan}\left[x + \sqrt{1-x^2}\right] - \frac{1}{4} \text{ArcTanh}\left[x \sqrt{1-x^2}\right] - \frac{1}{8} \text{Log}\left[1-x^2+x^4\right]
\end{aligned}$$

Result (type 3, 1822 leaves):

$$\begin{aligned}
& x \text{ArcTan}\left[x + \sqrt{1-x^2}\right] + \\
& \frac{1}{16} \left( -8 \text{ArcSin}[x] + 2 \sqrt{2+2i\sqrt{3}} \text{ArcTan}\left[\left(\frac{(1+i\sqrt{3}-2x^2)(-1+x^2)}{(-3i-\sqrt{3}+2\sqrt{3}x^4+x^3(-6-2i\sqrt{3}-2\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2}))}\right)\right] + \right. \\
& \quad \left. x \left(6+2i\sqrt{3}-2\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2}\right) + x^2 \left(3i-\sqrt{3}+2\sqrt{6-6i\sqrt{3}}\sqrt{1-x^2}\right)\right] - \\
& \quad 2 \sqrt{2+2i\sqrt{3}} \text{ArcTan}\left[\left(\frac{(1+i\sqrt{3}-2x^2)(-1+x^2)}{(-3i-\sqrt{3}+2\sqrt{3}x^4+2x(-3-i\sqrt{3}+\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2}))}\right)\right] + \\
& \quad \left. 2x^3 \left(3+i\sqrt{3}+\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2}\right) + x^2 \left(3i-\sqrt{3}+2\sqrt{6-6i\sqrt{3}}\sqrt{1-x^2}\right)\right] - \\
& \quad 2 \sqrt{2-2i\sqrt{3}} \text{ArcTan}\left[\left(\frac{(-1+x^2)(-1+i\sqrt{3}+2x^2)}{(3i-\sqrt{3}+2\sqrt{3}x^4+x(6-2i\sqrt{3}-2\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}))}\right)\right] + \\
& \quad \left. x^3 \left(-6+2i\sqrt{3}-2\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}\right) + x^2 \left(-3i-\sqrt{3}+2\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}\right)\right] + \\
& \quad 2 \sqrt{2-2i\sqrt{3}} \text{ArcTan}\left[\left(\frac{(-1+x^2)(-1+i\sqrt{3}+2x^2)}{(3i-\sqrt{3}+2\sqrt{3}x^4+2x^3(3-i\sqrt{3}+\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}))}\right)\right] +
\end{aligned}$$

$$\begin{aligned}
& 2x \left( -3 + i\sqrt{3} + \sqrt{2 + 2i\sqrt{3}} \sqrt{1-x^2} \right) + x^2 \left( -3i - \sqrt{3} + 2\sqrt{6 + 6i\sqrt{3}} \sqrt{1-x^2} \right) \Big] - \\
& 2 \operatorname{Log} \left[ -\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2 \right] + 2i\sqrt{3} \operatorname{Log} \left[ -\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2 \right] - 2 \operatorname{Log} \left[ \frac{1}{2} i (i + \sqrt{3}) + x^2 \right] - 2i\sqrt{3} \operatorname{Log} \left[ \frac{1}{2} i (i + \sqrt{3}) + x^2 \right] - \\
& i\sqrt{2 - 2i\sqrt{3}} \operatorname{Log} [16(1 + \sqrt{3}x + x^2)^2] + i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log} [16(1 + \sqrt{3}x + x^2)^2] + \\
& i\sqrt{2 - 2i\sqrt{3}} \operatorname{Log} [(4 - 4\sqrt{3}x + 4x^2)^2] - i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log} [(4 - 4\sqrt{3}x + 4x^2)^2] - \\
& i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log} [3i + \sqrt{3} - (-i + \sqrt{3})x^4 + 2i\sqrt{2 - 2i\sqrt{3}} \sqrt{1-x^2} + 5ix^2 (2 + \sqrt{2 - 2i\sqrt{3}} \sqrt{1-x^2}) + \\
& x (3 + 5i\sqrt{3} + 3i\sqrt{6 - 6i\sqrt{3}} \sqrt{1-x^2}) + ix^3 (3i + 3\sqrt{3} + \sqrt{6 - 6i\sqrt{3}} \sqrt{1-x^2})] + \\
& i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log} [3i + \sqrt{3} - (-i + \sqrt{3})x^4 + 2i\sqrt{2 - 2i\sqrt{3}} \sqrt{1-x^2} + 5ix^2 (2 + \sqrt{2 - 2i\sqrt{3}} \sqrt{1-x^2}) + \\
& x^3 (3 - 3i\sqrt{3} - i\sqrt{6 - 6i\sqrt{3}} \sqrt{1-x^2}) - ix ( -3i + 5\sqrt{3} + 3\sqrt{6 - 6i\sqrt{3}} \sqrt{1-x^2})] + \\
& i\sqrt{2 - 2i\sqrt{3}} \operatorname{Log} [-3i + \sqrt{3} - (i + \sqrt{3})x^4 - 2i\sqrt{2 + 2i\sqrt{3}} \sqrt{1-x^2} - 5ix^2 (2 + \sqrt{2 + 2i\sqrt{3}} \sqrt{1-x^2}) + \\
& x (3 - 5i\sqrt{3} - 3i\sqrt{6 + 6i\sqrt{3}} \sqrt{1-x^2}) - ix^3 ( -3i + 3\sqrt{3} + \sqrt{6 + 6i\sqrt{3}} \sqrt{1-x^2})] - \\
& i\sqrt{2 - 2i\sqrt{3}} \operatorname{Log} [-3i + \sqrt{3} - (i + \sqrt{3})x^4 - 2i\sqrt{2 + 2i\sqrt{3}} \sqrt{1-x^2} - 5ix^2 (2 + \sqrt{2 + 2i\sqrt{3}} \sqrt{1-x^2}) + \\
& x^3 (3 + 3i\sqrt{3} + i\sqrt{6 + 6i\sqrt{3}} \sqrt{1-x^2}) + ix (3i + 5\sqrt{3} + 3\sqrt{6 + 6i\sqrt{3}} \sqrt{1-x^2})] \Big]
\end{aligned}$$

**Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{x \operatorname{ArcTan} [x + \sqrt{1-x^2}]}{\sqrt{1-x^2}} dx$$

Optimal (type 3, 152 leaves, ? steps):

$$\begin{aligned}
& -\frac{\operatorname{ArcSin}[x]}{2} + \frac{1}{4}\sqrt{3} \operatorname{ArcTan} \left[ \frac{-1 + \sqrt{3}x}{\sqrt{1-x^2}} \right] + \frac{1}{4}\sqrt{3} \operatorname{ArcTan} \left[ \frac{1 + \sqrt{3}x}{\sqrt{1-x^2}} \right] - \\
& \frac{1}{4}\sqrt{3} \operatorname{ArcTan} \left[ \frac{-1 + 2x^2}{\sqrt{3}} \right] - \sqrt{1-x^2} \operatorname{ArcTan} [x + \sqrt{1-x^2}] + \frac{1}{4} \operatorname{ArcTanh} [x \sqrt{1-x^2}] + \frac{1}{8} \operatorname{Log} [1-x^2 + x^4]
\end{aligned}$$

Result (type 3, 2408 leaves):

$$-\frac{\text{ArcSin}[x]}{2} - \sqrt{1-x^2} \text{ArcTan}\left[x + \sqrt{1-x^2}\right] + \frac{1}{4\sqrt{6(1-i\sqrt{3})}}$$

$$\begin{aligned} &(-3i + \sqrt{3}) \text{ArcTan}\left[\left(3 - i\sqrt{3} - 12ix + 4\sqrt{3}x - 12i\sqrt{3}x^2 - 12ix^3 - 4\sqrt{3}x^3 - 3x^4 - \right. \right. \\ &\quad \left. \left. i\sqrt{3}x^4 - 2i\sqrt{2(1-i\sqrt{3})}x\sqrt{1-x^2} - 2i\sqrt{6(1-i\sqrt{3})}x^2\sqrt{1-x^2} - 2i\sqrt{2(1-i\sqrt{3})}x^3\sqrt{1-x^2}\right)\right] / \\ &\left(i - \sqrt{3} - 6x + 6i\sqrt{3}x + 30ix^2 - 2\sqrt{3}x^2 + 6x^3 + 18i\sqrt{3}x^3 + 11ix^4 + 3\sqrt{3}x^4\right) - \frac{1}{4\sqrt{6(1-i\sqrt{3})}} \end{aligned}$$

$$\begin{aligned} &(-3i + \sqrt{3}) \text{ArcTan}\left[\left(3 - i\sqrt{3} + 12ix - 4\sqrt{3}x - 12i\sqrt{3}x^2 + 12ix^3 + 4\sqrt{3}x^3 - 3x^4 - i\sqrt{3}x^4 + \right. \right. \\ &\quad \left. \left. 2i\sqrt{2(1-i\sqrt{3})}x\sqrt{1-x^2} - 2i\sqrt{6(1-i\sqrt{3})}x^2\sqrt{1-x^2} + 2i\sqrt{2(1-i\sqrt{3})}x^3\sqrt{1-x^2}\right)\right] / \\ &\left(i - \sqrt{3} + 6x - 6i\sqrt{3}x + 30ix^2 - 2\sqrt{3}x^2 - 6x^3 - 18i\sqrt{3}x^3 + 11ix^4 + 3\sqrt{3}x^4\right) - \frac{1}{4\sqrt{6(1+i\sqrt{3})}} \end{aligned}$$

$$\begin{aligned} &(3i + \sqrt{3}) \text{ArcTan}\left[\left(-3 - i\sqrt{3} - 12ix - 4\sqrt{3}x - 12i\sqrt{3}x^2 - 12ix^3 + 4\sqrt{3}x^3 + 3x^4 - i\sqrt{3}x^4 - \right. \right. \\ &\quad \left. \left. 2i\sqrt{2(1+i\sqrt{3})}x\sqrt{1-x^2} - 2i\sqrt{6(1+i\sqrt{3})}x^2\sqrt{1-x^2} - 2i\sqrt{2(1+i\sqrt{3})}x^3\sqrt{1-x^2}\right)\right] / \\ &\left(-i - \sqrt{3} - 6x - 6i\sqrt{3}x - 30ix^2 - 2\sqrt{3}x^2 + 6x^3 - 18i\sqrt{3}x^3 - 11ix^4 + 3\sqrt{3}x^4\right) + \frac{1}{4\sqrt{6(1+i\sqrt{3})}} \end{aligned}$$

$$\begin{aligned} &(3i + \sqrt{3}) \text{ArcTan}\left[\left(-3 - i\sqrt{3} + 12ix + 4\sqrt{3}x - 12i\sqrt{3}x^2 + 12ix^3 - 4\sqrt{3}x^3 + 3x^4 - i\sqrt{3}x^4 + \right. \right. \\ &\quad \left. \left. 2i\sqrt{2(1+i\sqrt{3})}x\sqrt{1-x^2} - 2i\sqrt{6(1+i\sqrt{3})}x^2\sqrt{1-x^2} + 2i\sqrt{2(1+i\sqrt{3})}x^3\sqrt{1-x^2}\right)\right] / \\ &\left(-i - \sqrt{3} + 6x + 6i\sqrt{3}x - 30ix^2 - 2\sqrt{3}x^2 - 6x^3 + 18i\sqrt{3}x^3 - 11ix^4 + 3\sqrt{3}x^4\right) - \end{aligned}$$

$$\frac{i(-3i + \sqrt{3}) \operatorname{Log}\left[(-i + \sqrt{3} - 2x)^2 (i + \sqrt{3} - 2x)^2\right]}{8\sqrt{6(1 - i\sqrt{3})}} + \frac{i(3i + \sqrt{3}) \operatorname{Log}\left[(-i + \sqrt{3} - 2x)^2 (i + \sqrt{3} - 2x)^2\right]}{8\sqrt{6(1 + i\sqrt{3})}} +$$

$$\frac{i(-3i + \sqrt{3}) \operatorname{Log}\left[(-i + \sqrt{3} + 2x)^2 (i + \sqrt{3} + 2x)^2\right]}{8\sqrt{6(1 - i\sqrt{3})}} -$$

$$\frac{i(3i + \sqrt{3}) \operatorname{Log}\left[(-i + \sqrt{3} + 2x)^2 (i + \sqrt{3} + 2x)^2\right]}{8\sqrt{6(1 + i\sqrt{3})}} +$$

$$\frac{(3i + \sqrt{3}) \operatorname{Log}\left[-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2\right]}{8\sqrt{3}} +$$

$$\frac{(-3i + \sqrt{3}) \operatorname{Log}\left[-\frac{1}{2} + \frac{i\sqrt{3}}{2} + x^2\right]}{8\sqrt{3}} +$$

$$\frac{1}{8\sqrt{6(1 - i\sqrt{3})}}$$

$$i(-3i + \sqrt{3}) \operatorname{Log}\left[3i + \sqrt{3} - 3x - 5i\sqrt{3}x + 10ix^2 + 3x^3 - 3i\sqrt{3}x^3 + ix^4 - \sqrt{3}x^4 + 2i\sqrt{2(1 - i\sqrt{3})}\sqrt{1 - x^2} - \right. \\ \left. 3i\sqrt{6(1 - i\sqrt{3})}x\sqrt{1 - x^2} + 5i\sqrt{2(1 - i\sqrt{3})}x^2\sqrt{1 - x^2} - i\sqrt{6(1 - i\sqrt{3})}x^3\sqrt{1 - x^2}\right] - \frac{1}{8\sqrt{6(1 - i\sqrt{3})}}$$

$$i(-3i + \sqrt{3}) \operatorname{Log}\left[3i + \sqrt{3} + 3x + 5i\sqrt{3}x + 10ix^2 - 3x^3 + 3i\sqrt{3}x^3 + ix^4 - \sqrt{3}x^4 + 2i\sqrt{2(1 - i\sqrt{3})}\sqrt{1 - x^2} + \right. \\ \left. 3i\sqrt{6(1 - i\sqrt{3})}x\sqrt{1 - x^2} + 5i\sqrt{2(1 - i\sqrt{3})}x^2\sqrt{1 - x^2} + i\sqrt{6(1 - i\sqrt{3})}x^3\sqrt{1 - x^2}\right] + \frac{1}{8\sqrt{6(1 + i\sqrt{3})}}$$

$$i(3i + \sqrt{3}) \operatorname{Log}\left[-3i + \sqrt{3} + 3x - 5i\sqrt{3}x - 10ix^2 - 3x^3 - 3i\sqrt{3}x^3 - ix^4 - \sqrt{3}x^4 - 2i\sqrt{2(1 + i\sqrt{3})}\sqrt{1 - x^2} - \right.$$



$$\begin{aligned}
 & 3i \sqrt{6(1+i\sqrt{3})} x \sqrt{1-x^2} - 5i \sqrt{2(1+i\sqrt{3})} x^2 \sqrt{1-x^2} - i \sqrt{6(1+i\sqrt{3})} x^3 \sqrt{1-x^2} - \frac{1}{8\sqrt{6(1+i\sqrt{3})}} \\
 & i(3i + \sqrt{3}) \operatorname{Log}[-3i + \sqrt{3} - 3x + 5i\sqrt{3}x - 10ix^2 + 3x^3 + 3i\sqrt{3}x^3 - ix^4 - \sqrt{3}x^4 - 2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2} + \\
 & 3i\sqrt{6(1+i\sqrt{3})}x\sqrt{1-x^2} - 5i\sqrt{2(1+i\sqrt{3})}x^2\sqrt{1-x^2} + i\sqrt{6(1+i\sqrt{3})}x^3\sqrt{1-x^2}]
 \end{aligned}$$

**Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Log}[x + \sqrt{-1+x^2}]}{(1+x^2)^{3/2}} dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$-\frac{1}{2} \operatorname{ArcCosh}[x^2] + \frac{x \operatorname{Log}[x + \sqrt{-1+x^2}]}{\sqrt{1+x^2}}$$

Result (type 3, 89 leaves):

$$\frac{4x \operatorname{Log}[x + \sqrt{-1+x^2}] + \frac{\sqrt{-1+x^2}(1+x^2) \left( \operatorname{Log}\left[1 - \frac{x^2}{\sqrt{-1+x^2}}\right] - \operatorname{Log}\left[1 + \frac{x^2}{\sqrt{-1+x^2}}\right] \right)}{\sqrt{-1+x^4}}}{4\sqrt{1+x^2}}$$

**Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3 \operatorname{ArcSin}[x]}{\sqrt{1-x^4}} dx$$

Optimal (type 3, 38 leaves, 5 steps):

$$\frac{1}{4} x \sqrt{1+x^2} - \frac{1}{2} \sqrt{1-x^4} \operatorname{ArcSin}[x] + \frac{\operatorname{ArcSinh}[x]}{4}$$

Result (type 3, 85 leaves):

$$\frac{1}{4} \left( \frac{x \sqrt{1-x^4}}{\sqrt{1-x^2}} - 2\sqrt{1-x^4} \operatorname{ArcSin}[x] + \operatorname{Log}[1-x^2] - \operatorname{Log}[-x+x^3 + \sqrt{1-x^2}\sqrt{1-x^4}] \right)$$

**Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[x]}{1 + \text{Sin}[x]^2} dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 46 leaves):

$$\frac{i \left( \text{ArcTan}\left[\frac{-i + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right] - \text{ArcTan}\left[\frac{i + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right] \right)}{\sqrt{2}}$$

**Problem 38: Result unnecessarily involves higher level functions.**

$$\int \frac{1 + x^2}{(1 - x^2) \sqrt{1 + x^4}} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2} x}{\sqrt{1 + x^4}}\right]}{\sqrt{2}}$$

Result (type 4, 36 leaves):

$$(-1)^{1/4} \left( \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} x\right], -1\right] - 2 \text{EllipticPi}\left[i, \text{ArcSin}\left[(-1)^{3/4} x\right], -1\right] \right)$$

**Problem 39: Result unnecessarily involves higher level functions.**

$$\int \frac{1 - x^2}{(1 + x^2) \sqrt{1 + x^4}} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2} x}{\sqrt{1 + x^4}}\right]}{\sqrt{2}}$$

Result (type 4, 40 leaves):

$$(-1)^{1/4} \left( \text{EllipticF} \left[ i \text{ArcSinh} \left[ (-1)^{1/4} x \right], -1 \right] - 2 \text{EllipticPi} \left[ -i, i \text{ArcSinh} \left[ (-1)^{1/4} x \right], -1 \right] \right)$$

**Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \text{Log}[\text{Sin}[x]] \sqrt{1 + \text{Sin}[x]} \, dx$$

Optimal (type 3, 42 leaves, 6 steps):

$$-4 \text{ArcTanh} \left[ \frac{\text{Cos}[x]}{\sqrt{1 + \text{Sin}[x]}} \right] + \frac{4 \text{Cos}[x]}{\sqrt{1 + \text{Sin}[x]}} - \frac{2 \text{Cos}[x] \text{Log}[\text{Sin}[x]]}{\sqrt{1 + \text{Sin}[x]}}$$

Result (type 3, 87 leaves):

$$\frac{1}{\text{Cos} \left[ \frac{x}{2} \right] + \text{Sin} \left[ \frac{x}{2} \right]} \\ 2 \left( -\text{Log} \left[ 1 + \text{Cos} \left[ \frac{x}{2} \right] - \text{Sin} \left[ \frac{x}{2} \right] \right] + \text{Log} \left[ 1 - \text{Cos} \left[ \frac{x}{2} \right] + \text{Sin} \left[ \frac{x}{2} \right] \right] - \text{Cos} \left[ \frac{x}{2} \right] \left( -2 + \text{Log}[\text{Sin}[x]] \right) + \left( -2 + \text{Log}[\text{Sin}[x]] \right) \text{Sin} \left[ \frac{x}{2} \right] \right) \sqrt{1 + \text{Sin}[x]}$$

**Problem 44: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[x]}{\sqrt{1 - \text{Sin}[x]}^6} \, dx$$

Optimal (type 3, 39 leaves, 4 steps):

$$\frac{\text{ArcTanh} \left[ \frac{\sqrt{3} \text{Cos}[x] (1 + \text{Sin}[x]^2)}{2 \sqrt{1 - \text{Sin}[x]}^6} \right]}{2 \sqrt{3}}$$

Result (type 4, 5825 leaves):

$$- \left( (-1)^{3/4} \left( 3i + (1 + 2i) \sqrt{2} 3^{1/4} + (1 + 2i) \sqrt{3} + i \sqrt{2} 3^{3/4} \right) \right. \\ \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{1}{2} \sqrt{\frac{(1+i) \left( (2 + \sqrt{2} 3^{1/4}) (2 + \sqrt{3}) + (2 - i \sqrt{2} 3^{1/4}) \text{Tan} \left[ \frac{x}{2} \right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \text{Tan} \left[ \frac{x}{2} \right]^2}} \right], 8 - 4\sqrt{3} \right] - 2 \times 3^{1/4} (\sqrt{2} + 3^{1/4}) \text{EllipticPi} \left[ \right. \right.$$

$$\frac{6(-3)^{1/4} - 2(-3)^{3/4} + 4\sqrt{3}}{3 + 3\sqrt{2}3^{1/4} + (2-i)\sqrt{3} + \sqrt{2}3^{3/4}}, \text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}3^{1/4}\right)\left(2+\sqrt{3}\right) + \left(2-i\sqrt{2}3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2}}\right], 8-4\sqrt{3}\right]$$

$$\text{Sin}[x] \sqrt{\frac{2i-2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2}{(-i\sqrt{2}+3^{1/4})\left(2i+2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2\right)}} \left(2-2(-1)^{3/4}3^{1/4}-i\sqrt{3}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\sqrt{-\frac{(i\sqrt{2}+3^{1/4})\left(-i+2(-2i+\sqrt{3})\text{Tan}\left[\frac{x}{2}\right]^2-i\text{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2i+2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}} \Big/$$

$$\left(\sqrt{2}3^{1/4}\left((3+6i)\sqrt{2}+(6+6i)3^{1/4}+(2+2i)3^{3/4}+(3+2i)\sqrt{6}\right)\sqrt{1-\text{Sin}[x]^6}\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2\right.$$

$$\sqrt{\frac{1+8\text{Tan}\left[\frac{x}{2}\right]^2+30\text{Tan}\left[\frac{x}{2}\right]^4+8\text{Tan}\left[\frac{x}{2}\right]^6+\text{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}} \left(-\left(\left(-1\right)^{3/4}\sqrt{2}\left(\left(3i+(1+2i)\sqrt{2}3^{1/4}+(1+2i)\sqrt{3}+i\sqrt{2}3^{3/4}\right)\text{EllipticF}\left[\right.\right.\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}3^{1/4}\right)\left(2+\sqrt{3}\right) + \left(2-i\sqrt{2}3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2}}\right], 8-4\sqrt{3}\right] - 2\times 3^{1/4}\left(\sqrt{2}+3^{1/4}\right)\text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{6(-3)^{1/4} - 2(-3)^{3/4} + 4\sqrt{3}}{3 + 3\sqrt{2}3^{1/4} + (2-i)\sqrt{3} + \sqrt{2}3^{3/4}}, \text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}3^{1/4}\right)\left(2+\sqrt{3}\right) + \left(2-i\sqrt{2}3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2}}\right], 8-4\sqrt{3}\right]\right)\right)$$

$$\text{Sec}\left[\frac{x}{2}\right]^2 \text{Tan}\left[\frac{x}{2}\right] \sqrt{\frac{2i-2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2}{(-i\sqrt{2}+3^{1/4})\left(2i+2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2\right)}} \left(2-2(-1)^{3/4}3^{1/4}-i\sqrt{3}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\sqrt{-\frac{(i\sqrt{2}+3^{1/4})\left(-i+2(-2i+\sqrt{3})\text{Tan}\left[\frac{x}{2}\right]^2-i\text{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2i+2(-3)^{1/4}+\sqrt{3}+i\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}} \Big/ \left(3^{1/4}\left((3+6i)\sqrt{2}+(6+6i)3^{1/4}+(2+2i)3^{3/4}+(3+2i)\sqrt{6}\right)\right)$$

$$\begin{aligned}
 & \left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^2 \sqrt{\frac{1 + 8 \tan\left[\frac{x}{2}\right]^2 + 30 \tan\left[\frac{x}{2}\right]^4 + 8 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{\left(1 + \tan\left[\frac{x}{2}\right]^2\right)^4}} \right) + \\
 & \left( (-1)^{3/4} \sqrt{2} \left( 3i + (1 + 2i) \sqrt{2} 3^{1/4} + (1 + 2i) \sqrt{3} + i \sqrt{2} 3^{3/4} \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2} \right. \right. \right. \\
 & \left. \left. \left. \sqrt{\frac{(1+i) \left( (2 + \sqrt{2} 3^{1/4}) (2 + \sqrt{3}) + (2 - i \sqrt{2} 3^{1/4}) \tan\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2} \right]}, 8 - 4\sqrt{3} \right] - 2 \times 3^{1/4} (\sqrt{2} + 3^{1/4}) \text{EllipticPi}\left[ \right. \right. \\
 & \left. \left. \frac{6(-3)^{1/4} - 2(-3)^{3/4} + 4\sqrt{3}}{3 + 3\sqrt{2} 3^{1/4} + (2 - i) \sqrt{3} + \sqrt{2} 3^{3/4}}, \text{ArcSin}\left[\frac{1}{2} \sqrt{\frac{(1+i) \left( (2 + \sqrt{2} 3^{1/4}) (2 + \sqrt{3}) + (2 - i \sqrt{2} 3^{1/4}) \tan\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2} \right]}, 8 - 4\sqrt{3} \right] \right) \\
 & \text{Sec}\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] \sqrt{\frac{2i - 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2}{(-i \sqrt{2} + 3^{1/4}) (2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2)}} \left( 2 - 2(-1)^{3/4} 3^{1/4} - i \sqrt{3} + \tan\left[\frac{x}{2}\right]^2 \right)^2 \\
 & \sqrt{-\frac{(i \sqrt{2} + 3^{1/4}) (-i + 2(-2i + \sqrt{3}) \tan\left[\frac{x}{2}\right]^2 - i \tan\left[\frac{x}{2}\right]^4)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2)^2}} \right) / \\
 & \left( 3^{1/4} \left( (3 + 6i) \sqrt{2} + (6 + 6i) 3^{1/4} + (2 + 2i) 3^{3/4} + (3 + 2i) \sqrt{6} \right) \left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^3 \sqrt{\frac{1 + 8 \tan\left[\frac{x}{2}\right]^2 + 30 \tan\left[\frac{x}{2}\right]^4 + 8 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{\left(1 + \tan\left[\frac{x}{2}\right]^2\right)^4}} \right) - \\
 & \left( (-1)^{3/4} \left( 3i + (1 + 2i) \sqrt{2} 3^{1/4} + (1 + 2i) \sqrt{3} + i \sqrt{2} 3^{3/4} \right) \right. \\
 & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2} \sqrt{\frac{(1+i) \left( (2 + \sqrt{2} 3^{1/4}) (2 + \sqrt{3}) + (2 - i \sqrt{2} 3^{1/4}) \tan\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2} \right]}, 8 - 4\sqrt{3} \right] - 2 \times 3^{1/4} (\sqrt{2} + 3^{1/4}) \text{EllipticPi}\left[ \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{6(-3)^{1/4} - 2(-3)^{3/4} + 4\sqrt{3}}{3 + 3\sqrt{2}3^{1/4} + (2-i)\sqrt{3} + \sqrt{2}3^{3/4}}, \operatorname{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left((2+\sqrt{2}3^{1/4})(2+\sqrt{3}) + (2-i\sqrt{2}3^{1/4})\tan\left[\frac{x}{2}\right]^2\right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2}}\right], 8 - 4\sqrt{3}\right] \\
 & \left(2 - 2(-1)^{3/4}3^{1/4} - i\sqrt{3} + \tan\left[\frac{x}{2}\right]^2\right)^2 \sqrt{-\frac{(i\sqrt{2} + 3^{1/4})(-i + 2(-2i + \sqrt{3})\tan\left[\frac{x}{2}\right]^2 - i\tan\left[\frac{x}{2}\right]^4)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2)^2}} \right. \\
 & \left. \left(-\frac{i\operatorname{Sec}\left[\frac{x}{2}\right]^2\tan\left[\frac{x}{2}\right](2i - 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2)}{(-i\sqrt{2} + 3^{1/4})(2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2)^2} + \frac{i\operatorname{Sec}\left[\frac{x}{2}\right]^2\tan\left[\frac{x}{2}\right]}{(-i\sqrt{2} + 3^{1/4})(2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2)}\right)\right) / \\
 & \left(2\sqrt{2}3^{1/4}\left((3+6i)\sqrt{2} + (6+6i)3^{1/4} + (2+2i)3^{3/4} + (3+2i)\sqrt{6}\right)\sqrt{\frac{2i - 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2}{(-i\sqrt{2} + 3^{1/4})(2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2)}} \right. \\
 & \left. \left(1 + \tan\left[\frac{x}{2}\right]^2\right)^2 \sqrt{\frac{1 + 8\tan\left[\frac{x}{2}\right]^2 + 30\tan\left[\frac{x}{2}\right]^4 + 8\tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{(1 + \tan\left[\frac{x}{2}\right]^2)^4}}\right) - \left((-1)^{3/4}\left(3i + (1+2i)\sqrt{2}3^{1/4} + (1+2i)\sqrt{3} + i\sqrt{2}3^{3/4}\right)\right. \\
 & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left((2+\sqrt{2}3^{1/4})(2+\sqrt{3}) + (2-i\sqrt{2}3^{1/4})\tan\left[\frac{x}{2}\right]^2\right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2}}\right], 8 - 4\sqrt{3}\right] - 2 \times 3^{1/4}(\sqrt{2} + 3^{1/4})\operatorname{EllipticPi}\left[\right.\right. \\
 & \left. \left. \frac{6(-3)^{1/4} - 2(-3)^{3/4} + 4\sqrt{3}}{3 + 3\sqrt{2}3^{1/4} + (2-i)\sqrt{3} + \sqrt{2}3^{3/4}}, \operatorname{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left((2+\sqrt{2}3^{1/4})(2+\sqrt{3}) + (2-i\sqrt{2}3^{1/4})\tan\left[\frac{x}{2}\right]^2\right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2}}\right], 8 - 4\sqrt{3}\right]\right) \\
 & \sqrt{\frac{2i - 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2}{(-i\sqrt{2} + 3^{1/4})(2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2)}}\left(2 - 2(-1)^{3/4}3^{1/4} - i\sqrt{3} + \tan\left[\frac{x}{2}\right]^2\right)^2 \\
 & \left(-\frac{(i\sqrt{2} + 3^{1/4})(2(-2i + \sqrt{3})\operatorname{Sec}\left[\frac{x}{2}\right]^2\tan\left[\frac{x}{2}\right] - 2i\operatorname{Sec}\left[\frac{x}{2}\right]^2\tan\left[\frac{x}{2}\right]^3)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2)^2} + \right. \\
 & \left. \frac{2i(i\sqrt{2} + 3^{1/4})\operatorname{Sec}\left[\frac{x}{2}\right]^2\tan\left[\frac{x}{2}\right](-i + 2(-2i + \sqrt{3})\tan\left[\frac{x}{2}\right]^2 - i\tan\left[\frac{x}{2}\right]^4)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i\tan\left[\frac{x}{2}\right]^2)^3}\right)\right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left( 2\sqrt{2} 3^{1/4} \left( (3+6i)\sqrt{2} + (6+6i)3^{1/4} + (2+2i)3^{3/4} + (3+2i)\sqrt{6} \right) \left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^2 \right. \\
 & \left. \sqrt{-\frac{(i\sqrt{2} + 3^{1/4}) \left( -i + 2(-2i + \sqrt{3}) \tan\left[\frac{x}{2}\right]^2 - i \tan\left[\frac{x}{2}\right]^4 \right)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2)^2}} \right) \sqrt{\frac{1 + 8 \tan\left[\frac{x}{2}\right]^2 + 30 \tan\left[\frac{x}{2}\right]^4 + 8 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{\left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^4}} \right) + \\
 & \left( (-1)^{3/4} \left( (3i + (1+2i)\sqrt{2} 3^{1/4} + (1+2i)\sqrt{3} + i\sqrt{2} 3^{3/4}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{2} \right. \right. \right. \right. \\
 & \left. \left. \left. \sqrt{\frac{(1+i) \left( (2 + \sqrt{2} 3^{1/4}) (2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \tan\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2}} \right], 8 - 4\sqrt{3} \right] - 2 \times 3^{1/4} (\sqrt{2} + 3^{1/4}) \operatorname{EllipticPi}\left[ \right. \right. \right. \\
 & \left. \left. \left. \frac{6(-3)^{1/4} - 2(-3)^{3/4} + 4\sqrt{3}}{3 + 3\sqrt{2} 3^{1/4} + (2-i)\sqrt{3} + \sqrt{2} 3^{3/4}}, \operatorname{ArcSin}\left[\frac{1}{2} \sqrt{\frac{(1+i) \left( (2 + \sqrt{2} 3^{1/4}) (2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \tan\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2}} \right], 8 - 4\sqrt{3} \right] \right) \right) \\
 & \sqrt{\frac{2i - 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2}{(-i\sqrt{2} + 3^{1/4}) (2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2)^2}} \left( 2 - 2(-1)^{3/4} 3^{1/4} - i\sqrt{3} + \tan\left[\frac{x}{2}\right]^2 \right)^2 \\
 & \sqrt{-\frac{(i\sqrt{2} + 3^{1/4}) \left( -i + 2(-2i + \sqrt{3}) \tan\left[\frac{x}{2}\right]^2 - i \tan\left[\frac{x}{2}\right]^4 \right)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i \tan\left[\frac{x}{2}\right]^2)^2}} \right. \\
 & \left. \frac{8 \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + 60 \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]^3 + 24 \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]^5 + 4 \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]^7}{\left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^4} \right) - \\
 & \left. \frac{4 \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] \left( 1 + 8 \tan\left[\frac{x}{2}\right]^2 + 30 \tan\left[\frac{x}{2}\right]^4 + 8 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8 \right)}{\left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^5} \right) \left/ \left( 2\sqrt{2} 3^{1/4} \right. \right. \\
 & \left. \left. \left( (3+6i)\sqrt{2} + (6+6i)3^{1/4} + (2+2i)3^{3/4} + (3+2i)\sqrt{6} \right) \left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^2 \left( \frac{1 + 8 \tan\left[\frac{x}{2}\right]^2 + 30 \tan\left[\frac{x}{2}\right]^4 + 8 \tan\left[\frac{x}{2}\right]^6 + \tan\left[\frac{x}{2}\right]^8}{\left( 1 + \tan\left[\frac{x}{2}\right]^2 \right)^4} \right)^{3/2} \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( (-1)^{3/4} \sqrt{\frac{2i - 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2}{(-i\sqrt{2} + 3^{1/4})(2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2)}} \left(2 - 2(-1)^{3/4} 3^{1/4} - i\sqrt{3} + \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^2 \right. \\
 & \sqrt{-\frac{(i\sqrt{2} + 3^{1/4})(-i + 2(-2i + \sqrt{3}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 - i \operatorname{Tan}\left[\frac{x}{2}\right]^4)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2)^2}} \left( \left( (3i + (1 + 2i)\sqrt{2} 3^{1/4} + (1 + 2i)\sqrt{3} + i\sqrt{2} 3^{3/4}) \right. \right. \\
 & \left. \left. \frac{(1+i)(2-i\sqrt{2} 3^{1/4}) \operatorname{Sec}\left[\frac{x}{2}\right]^2 \operatorname{Tan}\left[\frac{x}{2}\right] + (1-i) \operatorname{Sec}\left[\frac{x}{2}\right]^2 \operatorname{Tan}\left[\frac{x}{2}\right] \left( (2 + \sqrt{2} 3^{1/4})(2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 \right)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2)^2} \right) \right) \Bigg/ \\
 & \left( 4 \sqrt{\frac{(1+i) \left( (2 + \sqrt{2} 3^{1/4})(2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2}} \right. \\
 & \sqrt{1 - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left( (2 + \sqrt{2} 3^{1/4})(2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2}} \\
 & \left. \sqrt{1 - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (8 - 4\sqrt{3}) \left( (2 + \sqrt{2} 3^{1/4})(2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2}} \right) - \left( 3^{1/4} (\sqrt{2} + 3^{1/4}) \right. \\
 & \left. \frac{(1+i)(2-i\sqrt{2} 3^{1/4}) \operatorname{Sec}\left[\frac{x}{2}\right]^2 \operatorname{Tan}\left[\frac{x}{2}\right] + (1-i) \operatorname{Sec}\left[\frac{x}{2}\right]^2 \operatorname{Tan}\left[\frac{x}{2}\right] \left( (2 + \sqrt{2} 3^{1/4})(2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 \right)}{(2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2)^2} \right) \Bigg/ \\
 & \left( 2 \sqrt{\frac{(1+i) \left( (2 + \sqrt{2} 3^{1/4})(2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2}} \right. \\
 & \sqrt{1 - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left( (2 + \sqrt{2} 3^{1/4})(2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2}} \\
 & \left. \sqrt{1 - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (8 - 4\sqrt{3}) \left( (2 + \sqrt{2} 3^{1/4})(2 + \sqrt{3}) + (2 - i\sqrt{2} 3^{1/4}) \operatorname{Tan}\left[\frac{x}{2}\right]^2 \right)}{2i + 2(-3)^{1/4} + \sqrt{3} + i \operatorname{Tan}\left[\frac{x}{2}\right]^2}} \right)
 \end{aligned}$$



$$\left( 1 - \left( \left( \frac{1}{4} + \frac{i}{4} \right) \left( 6 (-3)^{1/4} - 2 (-3)^{3/4} + 4 \sqrt{3} \right) \left( \left( 2 + \sqrt{2} 3^{1/4} \right) \left( 2 + \sqrt{3} \right) + \left( 2 - i \sqrt{2} 3^{1/4} \right) \operatorname{Tan} \left[ \frac{x}{2} \right]^2 \right) \right) \right) /$$

$$\left( \left( \left( 3 + 3 \sqrt{2} 3^{1/4} + (2 - i) \sqrt{3} + \sqrt{2} 3^{3/4} \right) \left( 2 i + 2 (-3)^{1/4} + \sqrt{3} + i \operatorname{Tan} \left[ \frac{x}{2} \right]^2 \right) \right) \right) / \left( \sqrt{2} 3^{1/4} \right.$$

$$\left. \left( \left( (3 + 6 i) \sqrt{2} + (6 + 6 i) 3^{1/4} + (2 + 2 i) 3^{3/4} + (3 + 2 i) \sqrt{6} \right) \left( 1 + \operatorname{Tan} \left[ \frac{x}{2} \right]^2 \right)^2 \sqrt{\frac{1 + 8 \operatorname{Tan} \left[ \frac{x}{2} \right]^2 + 30 \operatorname{Tan} \left[ \frac{x}{2} \right]^4 + 8 \operatorname{Tan} \left[ \frac{x}{2} \right]^6 + \operatorname{Tan} \left[ \frac{x}{2} \right]^8}{\left( 1 + \operatorname{Tan} \left[ \frac{x}{2} \right]^2 \right)^4}} \right) \right) \right)$$

**Problem 47: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{ArcTan} \left[ x \sqrt{1 + x^2} \right] dx$$

Optimal (type 3, 120 leaves, 12 steps):

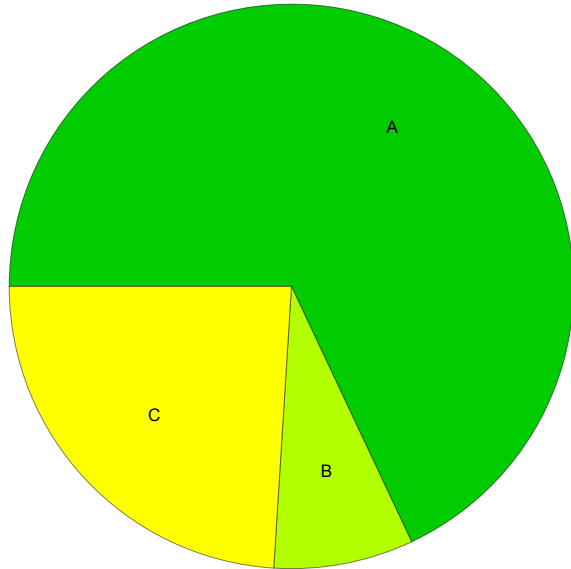
$$x \operatorname{ArcTan} \left[ x \sqrt{1 + x^2} \right] + \frac{1}{2} \operatorname{ArcTan} \left[ \sqrt{3} - 2 \sqrt{1 + x^2} \right] - \frac{1}{2} \operatorname{ArcTan} \left[ \sqrt{3} + 2 \sqrt{1 + x^2} \right] - \frac{1}{4} \sqrt{3} \operatorname{Log} \left[ 2 + x^2 - \sqrt{3} \sqrt{1 + x^2} \right] + \frac{1}{4} \sqrt{3} \operatorname{Log} \left[ 2 + x^2 + \sqrt{3} \sqrt{1 + x^2} \right]$$

Result (type 3, 116 leaves):

$$\frac{1}{2} \left( -\sqrt{-2 + 2 i \sqrt{3}} \operatorname{ArcTan} \left[ \frac{\sqrt{2} \sqrt{1 + x^2}}{\sqrt{-1 - i \sqrt{3}}} \right] - \sqrt{-2 - 2 i \sqrt{3}} \operatorname{ArcTan} \left[ \frac{\sqrt{2} \sqrt{1 + x^2}}{\sqrt{-1 + i \sqrt{3}}} \right] + 2 x \operatorname{ArcTan} \left[ x \sqrt{1 + x^2} \right] \right)$$

## Summary of Integration Test Results

50 integration problems



A - 34 optimal antiderivatives

B - 4 more than twice size of optimal antiderivatives

C - 12 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts